Exam. Code : 103204
Subject Code : 1110

B.A./B.Sc. 4th Semester

MATHEMATICS (Solid Geometry)

Paper—II

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt any **FIVE** questions, selecting at least **TWO** questions from each section.

SECTION—A

1. (a) Find the equation of the right circular cylinder whose axis is $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z}{3}$ and passes through (0, 0, 3). 5

   (b) Find the equation of parabolic cylinder whose generator is parallel to the z-axis and which intersects the parabola $y^2 = 4ax$, $z = 0$. 5

2. (a) Obtain the equation of right circular cylinder described on the circle through the points (a, 0, 0), (0, a, 0), (0, 0, a) as the guiding circle. 5

   (b) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$ and whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$. 5

3. (a) Find the equation of the cone with vertex at origin and which passes through the curve given by : $x^2 + y^2 + z^2 + x - 2y + 3z - 4 = 0$ and $x^2 + y^2 + z^2 + 2x - 3y + 4z - 5 = 0$. 5

(b) Show that $33x^2 + 13y^2 - 95z^2 - 144yz - 96zx - 48xy = 0$ represents a right circular cone whose axis is the line $3x = 2y = z$. Also find the vertical angle. 5

4. (a) The section of a cone whose vertex is P and guiding curve the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of P is $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$. 5

(b) Show that $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone. Also find the coordinates of vertex of this cone. 5

5. (a) Find the condition that the plane $lx + my + nz = 0$ may touch the cone $2x^2 - 3y^2 + z^2 = 0$ and find the equation of the reciprocal cone. 5

(b) Find the angle between the lines of sections of the following planes and cones :

$$3x + y + 5z = 0, \; 6yz - 2zx + 5xy = 0.$$ 5

SECTION—B

6. Reduce $3x^2 - y^2 - z^2 + 6yz - 6x + 6y - 2z - 2 = 0$ to standard form. Also find its centre and equation referred to center as origin. 10

7. (a) Write down the equation of the surface of revolution obtained by rotating the curve $y^2 + 16z^2 = 4$, $x = 0$ about the z-axis. 5

   (b) Reduce the equation $9x^2 + 4y^2 + 4z^2 + 8yz + 12zx + 12xy + 4x + y + 10z + 1 = 0$. 5

8. (a) Identify the curve $3x^2 + 2y^2 - 30x - 16y - 6z + 107 = 0$. 5

   (b) The normal at a point P on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the principal planes in G_1, G_2, G_3. If $PG_1^2 + PG_2^2 + PG_3^2 = k^2$, find the locus of P. 5

9. (a) Show that the plane $2x - 4y - z = 3$ touches the paraboloid $x^2 - 2y^2 = 3z$ and find the coordinates of the point of contact. 5

   (b) Find the equation of the tangent plane at the point (x_1, y_1, z_1) of the central conicoid $ax^2 + by^2 + cz^2 = 1$. 5

10. Prove that $5x^2 - 16y^2 + 5z^2 + 8yz - 14zx + 8xy + 4x + 20y + 4z - 24 = 0$ represents hyperbolic paraboloid. 10